General Physics

MAXIMAL LYAPUNOV EXPONENTS OF AN IMPACT OSCILLATOR

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An impact oscillator is essentially a system driven with a periodic forcing function, such as a ball bouncing on a table driven by a sinusoidally varying voltage. Fermi [1] suggested the impact oscillator as a model of cosmic rays interacting with magnetic fields. Other systems that are essentially impact oscillators include shock absorption systems, dot matrix printer heads, and articulated mooring towers, for example. Biological systems may include stereocilia. In each of these examples a system responds nonlinearly to a periodic driving force. In this work, we employ an electronic circuit that appears very much like a ball bouncing on an oscillating table as the impact oscillator [2]. Trajectories of the electronic ball are recorded on computer for analysis.

The analysis in this talk is limited to the maximal Lyapunov exponent, which is a measure of the rate of divergence of two trajectories with nearby initial conditions. The experimental results are compared to the results of a computer simulation of the system. The Lyapunov exponent can be calculated by

$$\lambda = \frac{1}{\Delta t} \sum_{i=1}^{m} \ln \frac{L'(t_i)}{L(t_{i-1})}, \tag{1}$$

where Δt is the time interval during which the system evolves and is fixed at 10 ms in this work, $L(t_i) = |\boldsymbol{X}_k - \boldsymbol{X}_l|$ is the initial separation between nearby vectors $\boldsymbol{X}_k$ and $\boldsymbol{X}_l$ defined by eq. (1) with $\boldsymbol{X}_k$ on the fiducial trajectory, and $L'(t_i) = |\boldsymbol{X}_{k'} - \boldsymbol{X}_{l'}|$ is the final separation between the trajectories that have evolved from their initial values. The summation is carried out for a total propagation time of 10 s for 10000 vectors using the same component spacing as for the correlation dimension calculation of T = 1 ms, but the spacing between the same components of adjacent vectors is only 1 ms. The calculation is based on the algorithm supplied by Wolf et al. [3]. In this case only a time series is required for a system where transients have died out and the trajectory is on an attractor.

We plan to compare these results to an alternative calculation of the maximal Lyapunov exponent, based on trajectories originating from nearby initial conditions in which the ball starts out at rest from various heights above the oscillating table. In this situation the ball is dropped at a certain table phase. The Lyapunov exponent is then obtained from the first term in eq. (1).

[1] E. Fermi, Phys. Rev. 75, (1949) 1169.
[2] B. K. Clark, R. F. Martin, Jr., R. J. Moore, and K. E. Jesse, Am. J. Phys. **63** (1995) 157.
[3] A. Wolf, J. B. Swift, H. L. Swinney, and J. A. Vastno, Physica D **16**, 285 (1985).